Sol^n of First and Second order network:

The circuit that contain capacitors and Inductors can be represented by differtial equations. If a circuit contain one resistor and one inductor (or one inductor) it can be represented by a first order differtial equation. If the circuit contain resistor, inductor and capacitor it canbe represented by a second order differtial equation. That is the order of the differtial equation of a circuit depends on the number of different energy storage elements that it contains. The sol^n of the differtial equation represents the response of the circuit.

In general the response consists of two parts.

① Natural response (Transient Response)

② Forced response (Steady state response)

Total response = Transient response + steady state response

or

= Natural response + forced response

Transient response: The value of Voltage and current during the transient period are known as transient response or natural response

Steady state response: The value of voltage and current after the transient has died out are known as steady state response or forced response.

Differential Equation:

Type-1 (First order Homogeneous differential Equation)

$$\frac{dy(t)}{dt} + Py(t) = 0$$

$$\Rightarrow \frac{dy(t)}{dt} = -Py(t)$$

$$\Rightarrow \ln y(t) = -Pt + K$$

$$K = \ln k$$

$$\Rightarrow \ln y(t) = -Pt + \ln k$$

$$y(t) = k\, e^{-Pt}$$

sol^n of diff^n. eqn,

$$\boxed{y(t) = Ke^{-Pt}}$$

Type-II (First order Non homogenous Diff^n eqn)

$$\frac{dy(t)}{dt} + Py(t) = Q.$$

sol^n of the above eqn diff^n.

$$\boxed{y(t) = \frac{Q}{P} + Ke^{-Pt}}$$

Type-III (Second order diff^n equation)

$$A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + C\,y(t) = 0$$

sol^n of the above eqn.

$$\boxed{y(t) = K_1e^{P_1t} + K_2e^{P_2t}}$$

where P_1, P_2 are the roots of the quadratic eq

$$AP^2 + BP + C = 0$$

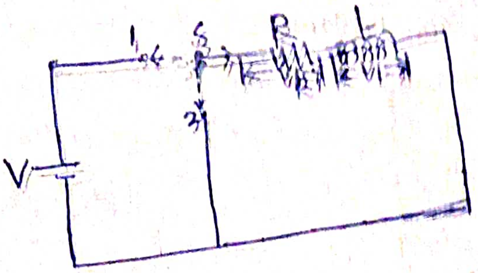
$$P_1, P_2 = -\frac{B}{2A} \pm \frac{1}{2A}\sqrt{B^2 - 4AC}$$

If $P_1 = P_2$, roots of the quadratic equation are repeated. Then the general solⁿ of the 2nd order diffⁿ eqⁿ.

$$\boxed{y(t) = K_1 e^{P_1 t} + K_2 t e^{P_1 t}}$$

GROWTH OF CURRENT IN AN INDUCTIVE CIRCUIT (D.C Transient)

(1)



Let us consider that a circuit having a resistance R and an inductance L connected to a d.c voltage V as shown. The R-L combination becomes connected to d.c voltage source when switch S is connected to terminal '1' and is short circuited when 'S' is connected to terminal '2'

When 'S' is connected to '1', the R-L combination is suddenly put across the voltage of V volt. Let us take the instant of closing S as the starting time zero. It is found that current does not reach its max^n value instantaneously but takes some finite time.

We will now investigate the growth of current through such an inductive ckt.

So According to Kirchhoff's voltage law

Total voltage applied = voltage across R + voltage to overcome self induced vol

$$V = V_R + V_L$$

$$V = iR + L\frac{di}{dt}$$

$$V - iR = L\frac{di}{dt}$$

$$\Rightarrow \frac{di}{V-iR} = \frac{dt}{L}$$

~~Integrating both side~~,

$$\int \frac{di}{V-iR} = \int \frac{dt}{L}$$

Multiplying both sides by (-R), we get

$$\int (-R)\frac{di}{V-iR} = \frac{-R}{L}dt$$

Integrating the both sides,

$$\int (-R)\frac{di}{V-iR} = \int \frac{-R}{L}dt$$

$\Rightarrow \log_e(V - iR) = -\frac{R}{L}t + K$

Where K is constant of integration whose value can be f from the initial known conditions.

When $t=0$, $i=0$,

$\log_e V = K$

$\therefore \log_e(V - iR) = -\frac{R}{L}t + \log V$

$\Rightarrow \log_e \frac{(V - iR)}{V} = -\frac{R}{L}t$

$\Rightarrow V - iR = V e^{-\frac{R}{L}t}$

$V - V e^{-\frac{R}{L}t} = iR$

$i = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$

$\boxed{i = \frac{V}{R}(1 - e^{-t/\tau})}$ where τ = time const = — (1)

So voltage across the resistor at any instant is

$V_R = iR$

$\Rightarrow \boxed{V_R = V(1 - e^{-t/\tau})}$

the voltage across the inductor L at any instant is

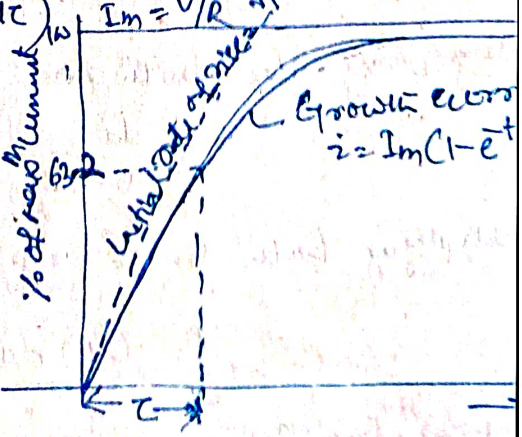
$V_L = V - V_R$

$= V - V(1 - e^{-t/\tau})$

$\boxed{V_L = V e^{-t/\tau}}$

Now $\frac{V}{R}$ represents the max^{m} steady value of curre

$i = I_m(1 - e^{-t/\tau})$



This is an exponential eqn whose graph as shown. It is seen that the current is rising rapid at first and then decreases until at $t = \infty$, it becomes zero. but theoretically current does ... the I_m until infinite time

... initial rate of rise of current $\frac{di}{dt}$ at any instant ...

Since $V = iR + L\frac{di}{dt}$

but at $t=0$, $i=0$, $V = 0 + L\frac{di}{dt}$ $\quad \frac{di}{dt} = \frac{V}{L}$

$\frac{di}{dt}$ = initial rate of rise $= \frac{V}{L}$

Final value of current

The current attains its final value after an infinite time i.e. when $t = \infty$

$$i = \frac{V}{R}(1 - e^{-t/\tau}) = \frac{V}{R}(1 - e^{-\infty}) = \frac{V}{R} - 0 = \frac{V}{R}$$

Thus the steady value of current $I_m = \frac{V}{R}$.

Now when time t = time const (τ)

then $\quad i = I_m(1 - e^{-\tau/\tau}) = I_m(1 - \frac{1}{e}) = I_m(1 - \frac{1}{2.718})$

$= 0.632 I_m$

Hence, the time const τ of an R-L ckt may also be defined as the time during which the current actually rises to 0.632 of its max^m steady value.

<u>Decay of current in an Inductive ckt</u>

When the switch 'S' is connected to point 2, the R-L ckt is short ckted. So initially when switch in position 1 for a long time the current in the R-L ckt to be max^m I_m. when the switch move to position 2, the circuit is disconnected from the supply and V becomes zero.

So the eqn for decay of current with time is found by putting $V = 0$

$$0 = iR + L\frac{di}{dt}$$

$$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides, we have

$$\int\frac{di}{i} = -\frac{R}{L}\int dt$$

$$\Rightarrow \log i = -\frac{R}{L}t + K$$

where K is integration const and it can be found out from initial known cond^n.

... the instant of switching off the ...
... is counted from this instant then $t = 0$.

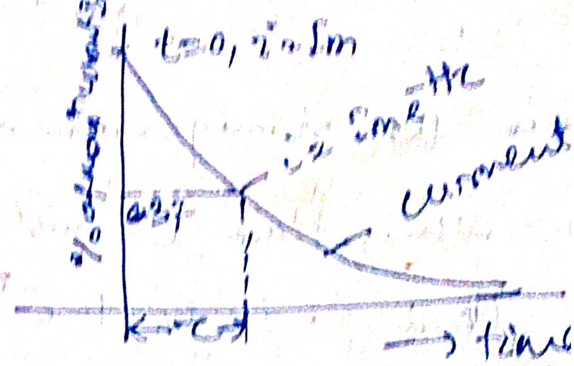
$$\log I_m = 0 + K \Rightarrow K = \log I_m$$

$$\therefore \log_e i = -\frac{R}{L}t + \log I_m \Rightarrow \log i - \log I_m = -\frac{R}{L}t$$

$$\Rightarrow \log \frac{i}{I_m} = -\frac{R}{L}t \Rightarrow i = I_m e^{-\frac{R}{L}t}$$

$$\boxed{i = I_m e^{-t/\tau}}$$

... is an decaying exponential function and its graph ... shown



When time $t = \tau$,

$$i = I_m e^{-\tau/\tau}$$
$$= I_m e^{-1} = \frac{I_m}{e}$$
$$= \frac{I_m}{2.718} = 0.37\, I_m$$

Hence time cont(τ) of an R-L ckt may also be defined as the time during which current falls to 0.37 or 37% of max^m steady value while decaying.

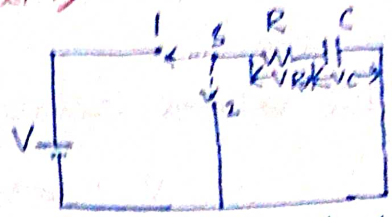
Transient State $(0 < t < \infty)$ → for growth of current.
It is also called instantaneous state i.e. any time ... closing or opening switch.

Initial State - just after closing the switch or just after opening the switch.

Steady State : i.e. a long time after closing or opening switch. In this state current in the conductor ... vary with time i.e. $\frac{dI}{dt} = 0$.

R-C (Growth and Decay in R-C ckt).

(Charging of the Capacitor)



Let us consider a circuit containing a capacitor C in series a resistance R connected to a dc supply at voltage V as shown.

When switch 'S' is connected to 1 C is charged but when it is connected to 2, C is short circuited through R and thus discharged. The voltage across C does not rise to V instantaneously but builds up slowly. Charging current is max^n at the start (i.e. when C is uncharged) then it gradually decreases.

Let V_R – voltage drop across R

V_C – voltage across C

q – charge on capacitor

i – charging current

Now applying the KVL,

$$V = V_R + V_C$$

$$V = iR + V_C \qquad \text{But } i = \frac{dq}{dt} \quad q = CV_C$$

$$= R\frac{d(CV_C)}{dt} + V_C$$

$$V - V_C = RC\frac{dV_C}{dt}$$

$$\frac{dV_C}{V - V_C} = \frac{dt}{RC}$$

Now integrating the both sides, we have

or multiplying '-1' in both $\int \frac{dV_C}{V-V_C} = -\frac{t}{RC}$

$$\int \frac{dV_C}{V - V_C} = \int \frac{dt}{RC}$$

$$-\log(V - V_C) = \frac{t}{RC} + K$$

where K is const of integration and it can be found out from the initial known cond^n.

When $t = 0$, $V_C = 0$

So $-\log V = K$

Now putting the K value in the above eq^n, we have

$$-\log(V - V_C) = \frac{t}{RC} - \log V$$

$$\log V - \log(V - V_C) = \frac{t}{RC}$$

$$\log\frac{V}{V - V_C} = \frac{t}{RC} \quad \text{or} \quad \log\left(\frac{V - V_C}{V}\right) = -\frac{t}{RC}$$

$$\frac{V - V_C}{V} = e^{-\frac{t}{RC}} \quad \text{or} \quad V - V_C = Ve^{-t/RC}$$

The above eqn gives the voltage across the ...
at any instant.

So the voltage across the resistor at any instant

$$v_R = V - v_c = V - V(1-e^{-t/RC}) = Ve^{-t/RC}$$

Now $v_c = \frac{q}{C}$ and $V = \frac{Q}{C}$

So $\frac{q}{C} = \frac{Q}{V}(1-e^{-t/RC})$

$q = Q(1-e^{-t/RC})$

So we found that increase of charge, like growth of potential which is an exponential function and its steady state is reached after infinite time.

Now $i = \frac{dq}{dt} = Q\frac{d}{dt}(1-e^{-t/RC})$

$= \left[\frac{Q}{RC}e^{-t/RC}\right] = \frac{CV}{RC}e^{-t/RC}$

$i = \frac{V}{R}e^{-t/RC} = I_m e^{-t/RC}$

Where I_m = max current $= \frac{V}{R}$

As charging continues, charging current decreases as from the above eqn. It becomes zero when $t = \infty$

Different cond^n in the RC growth ckt.

(i) At that time $t = 0$, just closing the switch, the p.d across the capacitor is zero. So $V_c = 0$.

$V = V_R + V_c$

$= RC\frac{dv_c}{dt} + 0$

$\frac{dv_c}{dt} = \frac{V}{RC}$

i.e. the initial rate of rise of voltage across the capacitor

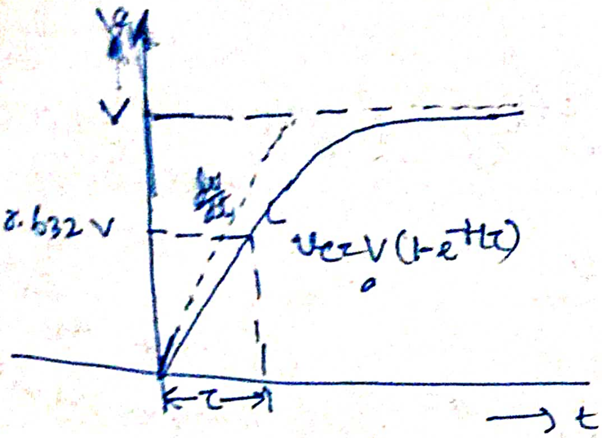
$= \frac{dv_c}{dt} = \frac{V}{RC}$

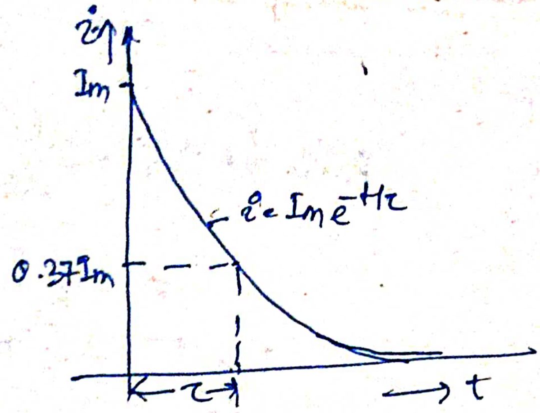
(ii) When time(t) = time const(τ) then   Time const = RC

$V_c = V(1-e^{-t/RC}) = V(1-e^{-\tau/\tau}) = V(1-e^{-1})$

... and Decay in R-C ckt).

(capacitor)





When $t=\infty$, $V_C = V(1-e^{-\infty/\tau}) = V$.

When $t=\tau$, $i = I_m e^{-t/\tau} = I_m e^{-1} = \frac{I_m}{2.718} = 0.37 I_m$

Decay of voltage (Discharge of capacitor)

When switch S is shifted to 2, the capacitor is discharg through R. To begin with discharge current is max^m but then decreases till it is fully discharged.

Since battery is cut out of the ckt, therefore V=0.

$$V = V_R + V_C$$
$$0 = iR + V_C$$
$$= R\frac{dq}{dt}R + V_C$$
$$= RC\frac{dV_C}{dt} + V_C$$
$$\Rightarrow V_C = -RC\frac{dV_C}{dt}$$
$$\frac{dV_C}{V_C} = -\frac{dt}{RC}$$

Integrating the both sides, we have

$$\int \frac{dV_C}{V_C} = -\int \frac{dt}{RC}$$
$$\log V_C = -\frac{t}{RC} + K$$

where K is const of integration, when $t=0$, $V_C = V$ (i.e., the $\left[\begin{matrix} V = iR + V_C \\ V = V_C \end{matrix}\right]$)

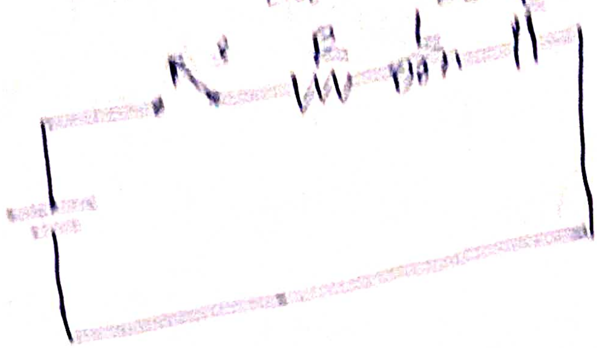
$$\log V = 0 + K$$
$$\log V_C = -\frac{t}{RC} + \log V$$
$$\log \frac{V_C}{V} = -\frac{t}{RC} \text{ or } \frac{V_C}{V} = e^{-\frac{t}{RC}}$$

$$V_C = Ve^{-\frac{t}{RC}} = Ve^{-t/\tau}$$ where τ = time const = RC

Similarly $q = Q\, e^{-t/\tau}$

...l Response of Series R-L-C Ckt with D.C Excitation

In a R-L-C series ckt having $V_s = 2V$, $R = 6\Omega$, $L = 2H$, $C = 0.25F$. Determine $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2i(0^+)}{dt^2}$ and $i(t)$



$... + i(t)R + \frac{1}{C}\int i(t)dt = V_s = 2$ —— (1)

...tiating the eqn (1),

$$= 2\frac{d^2i(t)}{dt^2} + 6\frac{di(t)}{dt} + \frac{1}{0.25}i(t)$$

$$\Rightarrow \frac{d^2i(t)}{dt^2} + 3\frac{di(t)}{dt} + 2i(t) = 0 \quad \text{—— (2)}$$

$$p^2 + 3p + 2 = 0$$

The roots of the above eqn,

$$p_1 = -1,\ p_2 = -2$$

The sol^n of eqn (2)

$$i(t) = K_1 e^{-t} + K_2 e^{-2t} \quad \text{—— (3)}$$

When Switch is closed at $t=0$, $i(0^+) = 0$

$$V_s = L\frac{di(t)}{dt}\Big|_{t=0} \Rightarrow \frac{di(0^+)}{dt} = \frac{V_s}{L} = \frac{2}{2} = 1$$

...n (2) $\frac{d^2i(t)}{dt^2}\Big|_{t=0} + 3\frac{di(t)}{dt}\Big|_{t=0} + 2i(0^+) = 0$

$$\frac{d^2i(0)}{dt^2} + 3\times 1 + 2\times 0 = 0 \qquad \frac{d^2i(0)}{dt^2} = -3\ A/sec^2$$

(3) $\quad 0 = K_1 + K_2$ —— (4)

$$\frac{di(t)}{dt} = -k_1 e^{-t} - 2k_2 e^{-2t}$$

$$\left.\frac{di(t)}{dt}\right|_{t=0} = -k_1 - 2k_2$$

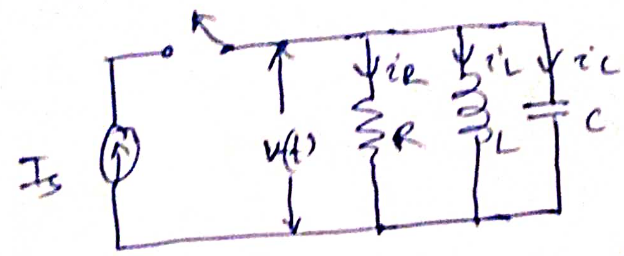
$$\Rightarrow \quad 1 = -k_1 - 2k_2 \quad \text{—} \quad (5)$$

From eqn (4) & (5) $\quad k_1 = 1, \; k_2 = -1$

$$i(t) = e^{-t} - e^{-2t}$$

Transient Response in RLC parallel ckt.

Ex:



In the above ckt. $I_s = 2A$, $R = \frac{1}{16}\,\Omega$, $L = \frac{1}{16}\,H$, $C = 4F$.

Determine $v(0^+)$. $\frac{dv(0^+)}{dt}$, $\frac{d^2v(0^+)}{dt^2}$ and $v(t)$

$$I_s = i_R + i_L + i_C$$

$$= \frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt + C\frac{dv(t)}{dt} \quad \text{—} \quad (1)$$

Differentiating the eqn (1).

$$0 = 16\frac{dv(t)}{dt} + 16\,v(t) + 4\frac{d^2v(t)}{dt^2}$$

$$\Rightarrow \quad \frac{d^2v(t)}{dt^2} + 4\frac{dv(t)}{dt} + 4\,v(t) = 0 \quad \text{—} \quad (2)$$

$$P^2 + 4P + 4 = 0$$

The roots of the eqn $\quad P_1 = -2, \; P_2 = -2$

The solⁿ of the eqⁿ ②.

$$v(t) = K_1 e^{-2t} + K_2 t e^{-2t} \quad \text{--- } (3)$$

at time $t = 0$, $v(0^+) = 0$ → as capacitor act short ckt

In eqⁿ (1)

$$2 = 0 + 0 + 4 \frac{dv(0^+)}{dt}$$

$$\Rightarrow \frac{dv(0^+)}{dt} = \frac{2}{4} = 1/2 \text{ V/sec.}$$

In eqⁿ ②
$$\frac{d^2v(0)}{dt^2} + 4\frac{dv(0)}{dt} + 4\,v(0^+) = 0$$

$$\Rightarrow \frac{d^2v(0)}{dt^2} + 4 \times \frac{1}{2} + 4 \times 0 = 0$$

$$\frac{d^2v(0)}{dt^2} = -2 \text{ V/sec}^2$$

In eqⁿ ③,
$$0 = K_1 + K_2 \times 0$$

$$K_1 = 0$$

Diffⁿ in eqⁿ ③ we get

$$-2K_1 + K_2 = 1/2$$

$$K_2 = 1/2$$

$$v(t) = 1/2\, t e^{-2t}$$

Q. A d.c voltage of 200V is suddenly applied series R-L ckt having $R = 20\Omega$, $L = 0.2H$. Determine voltage drop across the inductor at the instant switching on and 0.02 sec later.

$i(t) = \frac{V}{R}(1 - e^{-t/\tau})$ $\tau = \frac{L}{R} = \frac{0.2}{20} = \frac{1}{100}$

$i(t) = \frac{200}{20}(1 - e^{-0.02 \times 100})$

$= 8.646\,A$

$V = V_R + V_L \Rightarrow V_L = V - V_R = V - iR$

$V_L = 200 - 8.646 \times 20 = 27\,V$

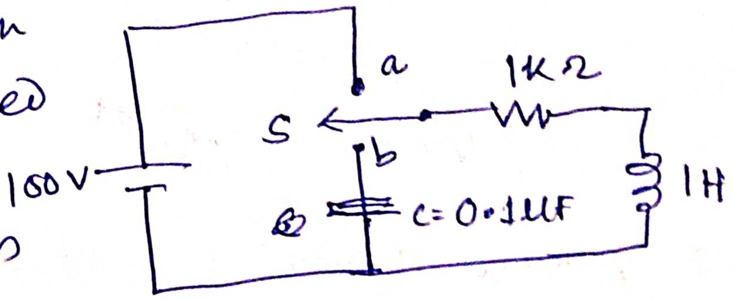
A resistance R and 5μF capacitor are connected in series across a 100 V D.C supply. Calculate the value of R such that the voltage across the capacitor become 50V in 5 sec after the circuit is switched on.

solⁿ: $V_C(t) = V(1 - e^{-t/\tau})$ $\tau = RC$

$50 = 100\left(1 - e^{-\frac{5}{R \times 5 \times 10^{-6}}}\right)$

$R = 1.45 \times 10^{6}\,\Omega.$

Q. In the ckt, when switch S is changed from position a to b at $t = 0$. Find values of i, $\frac{di}{dt}$ and $\frac{d^2 i}{dt^2}$ at $t = 0^+$



Sol: At position 'a',
The steady state value of current $i(0^-) = \frac{100}{1000} =$

position 'b'

$$1000\, i(t) + 1\frac{d\,i(t)}{dt} + \frac{1}{0.1\times10^{-6}}\int_{-\infty}^{t} i(t)\,dt = 0 \quad (1)$$

$t=0$ $\quad 1000\, i(0^+) + \frac{d\,i(0^+)}{dt} + 0 = 0 \qquad i(0^+) = 0.1$

$$1000 \times 0.1 + \frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = -100 \text{ A/sec.}$$

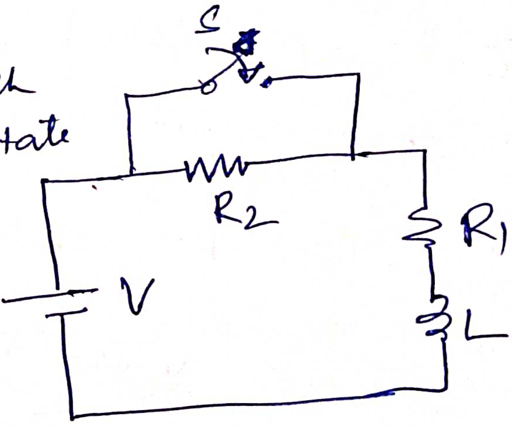
Diff.^{n} eqn (1) $\quad 1000\frac{di(t)}{dt} + 1\frac{d^2 i(t)}{dt^2} + \frac{i(t)}{0.1\times10^{-6}} = 0$

at $t=0$ $\quad 1000 \times \frac{di(0^+)}{dt} + \frac{d^2 i(0^+)}{dt^2} + \frac{i(0)}{0.1\times10^{-6}} = 0$

$$1000 \times (-100) + \frac{d^2 i(0)}{dt^2} + \frac{0.1}{0.1\times10^{-6}} = 0$$

$$\frac{d^2 i(0^+)}{dt^2} = 10^5 - \frac{1}{10^{-6}} = -9\times10^5 \text{ A/sec}^2$$

In the given ckt, when switch s is closed at $t=0$, a steady state current having previously been attained. solve for the current as a function of time.



Sol^n: Before switch action takes place

$$i(0) = \frac{V}{R_1+R_2}$$

when switch is closed.

$$V = L\frac{di}{dt} + R_1 i$$

$$\frac{di}{dt} + \frac{R_1}{L} i(t) = \frac{V}{L} \quad (2)$$

Sol^n of eq^n (2).

$$i(t) = \frac{V}{R_1} + K e^{-\frac{R_1}{L}t}$$

at time $t=0$, $\quad \frac{V}{R_1+R_2} = \frac{V}{R_1} + K$

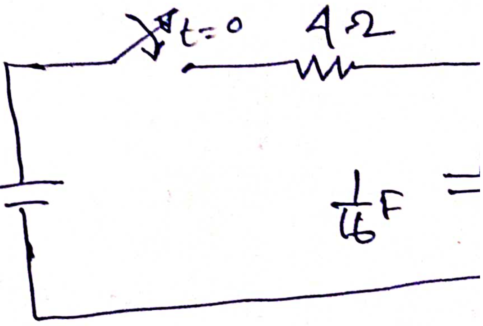
$i(0) = \frac{V}{R_1+R_2}$

$$K = \frac{V}{R_1+R_2} - \frac{V}{R_1} = -\frac{V \cdot R_2}{R_1(R_1+R_2)}$$

$$i(t) = \frac{V}{R_1} - \frac{V \cdot R_2}{R_1(R_1+R_2)} e^{-\frac{R_1}{L}t}$$

$$i(t) = \frac{V}{R_1}\left(1 - \frac{R_2}{R_1+R_2} e^{-\frac{R_1}{L}t}\right)$$

Q. In the given ckt. find the value of $v_c(t)$ for $t > 0$, Assume initial condition $v_c(0^-) = 9V$.



Sol^n:
$$1 = 4i(t) + v_c(t) \quad \text{— (a)}$$

$$v_c(t) = 9 + 16\int_0^t i(t)\,dt$$

$$1 = 4i(t) + 9 + 16\int_0^t i(t)\,dt$$

$$4i(t) + 16\int_0^t i\,dt = -8$$

$$i(t) + 4\int_0^t i(t)\,dt = -2 \quad \text{— (1)}$$

Diff^n eq^n (2) $\quad \frac{di(t)}{dt} + 4i(t) = 0 \quad$ — (3)

sol^n of the eq^n (3) $\quad i(t) = Ke^{-4t} \quad$ — (4)

at from eq^n (a) $\quad 1 = 4i(0^+) + 9 \Rightarrow i(0) =$

[redacted]

the impedance of some of the branches changes the

From Eqn (4) $\quad i(0^+) = k.$

$$\Rightarrow k = -2$$

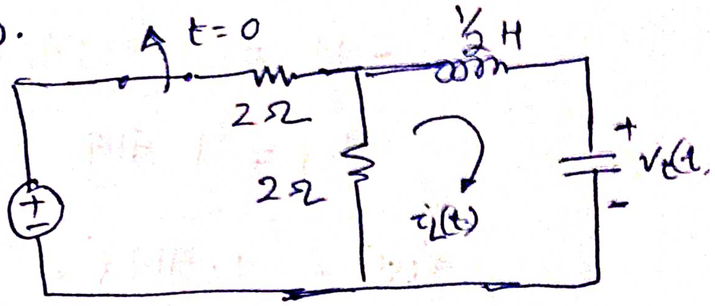
$$i(t) = -2e^{-4t}$$

$$v_c(t) = 9 + 16\int_0^t i(t)\,dt$$

$$= 9 + 16\int_0^t -2e^{-4t}dt = 9+16\left[\frac{-2e^{-4t}}{-4}\right]\Big|_{t=0}^{t}$$

$$v_c(t) = 9 + 8e^{-4t} - 8 = 1 + 8e^{-4t}\,V$$

Q: In the ckt as shown in fig. is in the steady state with the switch S closed. The switch is opened at $t = 0$. Determine $i_L(t)$ in the ckt.



Sol^n – At steady state with the switch S closed. The capacitor behaves as a open ckt, $v_c(0^-) = 2V$, and $i_L(0^-)$

When switch is opened. $\quad v_c(0^+) = v_c(0^-)$

and $\quad i_L(0^+) = i_L(0^-)$

$$2i_L + \frac{1}{2}\frac{di_L(t)}{dt} + \frac{1}{1}\int_0^t i_L(t) + v_c(0^-) = 0$$

Diff^n. the above eqn.

$$2\frac{di_L}{dt} + \frac{1}{2}\frac{d^2 i_L(t)}{dt^2} + i_L(t) = 0$$

$$\frac{d^2 i_L(t)}{dt^2} + 4\frac{di_L(t)}{dt} + i_L(t) = 0$$

C.S. Eqn. $\quad P^2 + 4P + 2 = 0 \quad , \Rightarrow \quad P_1 = -3.414$

$$P_2 = -0.586.$$

Sol^n of the above eq^n $i_L(t) = k_1 e^{-3.414t} + k_2 e^{-0.586t}$.

$i_L(0^-) = 0$ therefore $k_1 + k_2 = 0$

$V_L(0^+) = -V_c(0^+) = -2 = L\frac{di_L(0^+)}{dt}$

$$\frac{di_L(0^+)}{dt} = -\frac{2}{L} = \frac{-2}{1/2} = -4$$

$$\frac{di_L}{dt} = k_1 \times (-3.414) e^{-3.144t} + k_2(-0.586) \times e^{-0.586t}$$

$-4 = -3.141k_1 - 0.586k_2$.

$k_1 = 1.414, \; k_2 = -1.414$

$$i_L(t) = 1.414\left(e^{-3.414t} - e^{-0.586t}\right)$$